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AN ENERGY-BALANCE ANALYSIS OF THE  
INTERACTION BETWEEN CONDUCTING  
BODIES AND ELECTRIC FIELDS

William Bucher

April 1977

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) (bjk) The application of the principle of energy-balance is an especially useful approach to problems for which a complete analytical solution is not possible but for which partial information can provide a thorough understanding of system dynamics. In this report the problem of N conducting bodies immersed in the electric field produced by a pair of electrodes is examined, and associated energy-balance equations are derived. These equations are applicable to a wide variety of problems. A theorem on the partition of energy for electrically neutral bodies is also presented.		

## FOREWORD

This report was completed by Dr. William Bucher shortly before his death in an automobile accident while on his way to work on 11 Feb 77. Aside from small changes to update footnotes, it is exactly as he wrote it. Although Dr. Bucher was an experimental physicist in the classical sense, his fine capabilities as a theoretician are illustrated in this publication.

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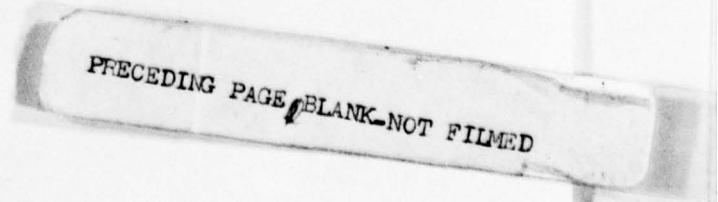
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## I. INTRODUCTION

A detailed energy-balance study offers an effective means for analyzing the motion of conducting bodies immersed in an electric field produced by a pair of electrodes. By considering the partition of energy throughout such a system, exact and highly manageable expressions can be derived to provide a description of the system dynamics. As the system moves from one state to another under the influence of electrostatic or external forces, changes occur in the physical quantities associated with the state of the system. Such changes determine completely the dynamics for conservative systems. For nonconservative systems, as will become apparent, the inclusion of dissipative forces (e.g., sparking, corona, etc.) can be treated in a straightforward way. Quantities of particular interest here are the kinetic energy, both rotational and translational, of the conductor; the electrostatic field energy; the electrical energy supplied to system by battery; energy dissipation; and current flow in the external circuit.

In this report, the parallel plate electrode configuration is examined in detail. For other electrode configurations, the appropriate energy partition equations can be derived in a manner similar to the development presented here. For example, spherical-electrode geometry, which is applicable to the particle detector of Shelton and Morrissey<sup>1</sup> was examined previously.<sup>2</sup> (See Reference 2 for the analyses of particle detection in both parallel plate and spherical geometries, and conductor link-up across electrode gaps.)

Following the derivation of the energy equations for charged and uncharged conducting bodies between parallel plate electrodes, a theorem on the partition of energy for electrically neutral bodies is presented. In addition, solutions to a variety of problems are presented which illustrate the range and utility of the energy-balance approach.

## II. ENERGY-BALANCE FORMULATION

Figure 1a shows schematically the geometry under consideration. While the shape of the single conductor depicted in this figure is rather specialized, it will be noted that no such restrictions apply in the following treatment. Indeed, configurations of two or more arbitrarily shaped conductors are also permitted, provided only that the individual surface charge distributions reflect the effects of all mutual interactions.

<sup>1</sup>R. D. Shelton and J. Morrissey, "Airborne Particle Analyzer," patent pending.

<sup>2</sup>W. Bucher, "On the Phenomenon of Current Pulses Generated by Conductors Near Contact in Electric Fields," BRL Report No. 1951, Dec 76.

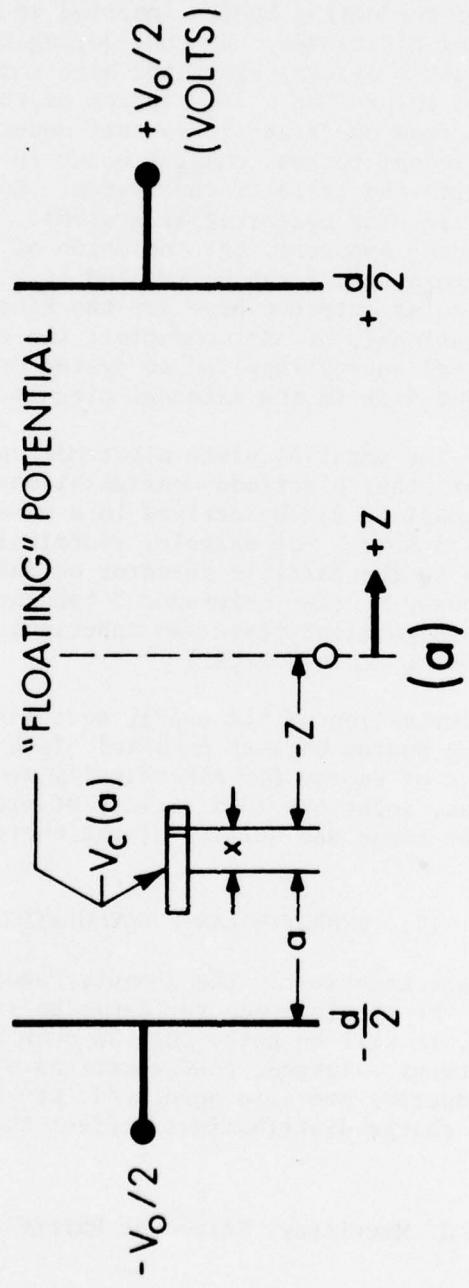


Figure 1a. Single Conductor in Parallel Plate Geometry

It is convenient to represent the total electrostatic energy  $\xi_a$  of Figure 1a as the sum,

$$\xi_a = \xi_b + \xi_c + \xi_{b,c}, \quad (1)$$

where  $\xi_b$  and  $\xi_c$  are the total energies of the subsystems of Figures 1b and 1c, respectively, and  $\xi_{b,c}$  is their interaction energy. For any charge distribution, the energy of assembly is given by the well known relation

$$U = \frac{1}{2} \sum_{j \neq i} \mathbf{S} v_i q_j + \sum_j w_{(self)j}, \quad (2)$$

where  $\mathbf{S}$  indicates the required summation and/or integration over  $j$ ;  $v_i$  is the potential at the point  $j$  due to all charges except  $q_j$ ; and  $w_{(self)j}$  is the self-energy of any point charge  $j$ . Since  $w_{(self)}$  is constant, it can be neglected as it will cancel in the final result, and thus

$$U = \frac{1}{2} \int V(r) dq \quad (3)$$

will be used to determine assembly energies. Similarly,  $\xi_c$  is constant and by its omission Equation (1) reduces to

$$\xi = \xi_b + \xi_{b,c}. \quad (4)$$

Applying Equation (3) yields

$$\xi_b = \frac{1}{2} (Q-) (0) + \frac{1}{2} (Q+) (0) + \frac{1}{2} \int_C v(\alpha, z) dq \quad (5)$$

where the region  $\int_C$  is over the surface of the conductor,

$$v(\alpha, z) = V(\alpha) - E_0 Z, \quad (6)$$

and  $V(\alpha)$  is the floating potential on the conductor. By defining  $q_0$  as the charge distribution the conductor would have if the net charge were zero, then  $q$  can be expressed as

$$q = q_{net} + q_0, \quad (7)$$

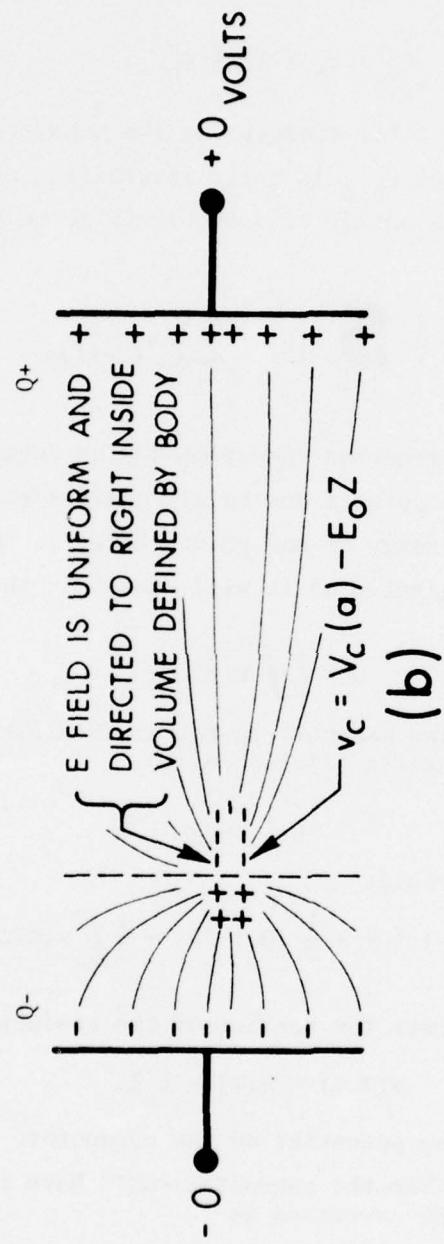


Figure 1b. Single Conductor in Parallel Plate Geometry

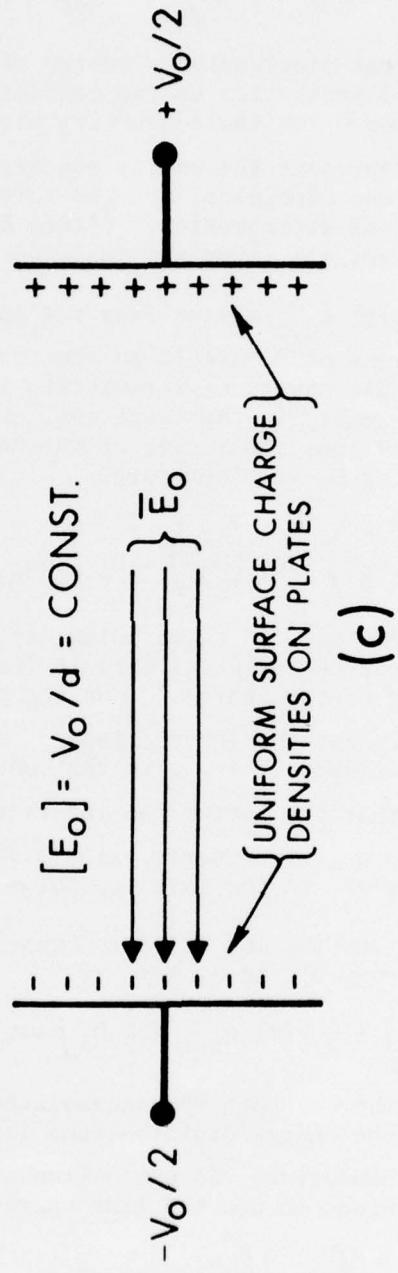


Figure 1c. Single Conductor in Parallel Plate Geometry

and Equation (5) can be rewritten as

$$\xi_b = \frac{1}{2} V(a) q_{\text{net}} - \frac{1}{2} E_0 \int_C z dq_{\text{net}} - \frac{1}{2} E_0 \int_C z dq_0. \quad (8)$$

It is noted that the total electrostatic energy of Figure 1b is associated with the charge distribution on the conductor; none is involved in assembling the charges  $Q_{\pm}$  on the conducting planes. The first two terms of Equation (8) represent the energy required to assemble the net charge distribution on the conductor, and the third term is the energy of assembly of the neutral distribution. (Since both integrals in Equation (8) are negative, all terms are positive.)

The interaction energy  $\xi_{b,c}$  arises from the superposition of the fixed charge distributions of Figure 1b on the constant field configuration of Figure 1c. This energy of interaction is basically distinguished from that of assembly in that with the former the charge distributions are fixed and thus the factor of one-half is omitted from Equation (3) in computing  $\xi_{b,c}$ . Therefore,

$$\xi_{b,c} = Q_- \left( \frac{-V_0}{2} \right) + Q_+ \left( \frac{+V_0}{2} \right) + E_0 \int z dq_{\text{net}} + E_0 \int z dq_0, \quad (9)$$

where the first two terms may, as shown below, be interpreted as the conversion of energy from the external circuit (battery) to the electrostatic potential energy of the charges  $Q_{\pm}$  on the conducting planes, and the last two terms represent the interactions of the fixed charged and neutral distributions, respectively, with the constant electric field  $\bar{E} = -E_0 \hat{z}$ . It is seen that the latter two interactions are attractive (both integrals lead to negative energy values) and further that they are proportional (times -2) to the last two terms of Equation (8).

Substituting Equations (8) and (9) into Equation (4) yields for the total electrostatic energy of the system

$$\xi = \frac{1}{2} V_0 \left( Q_+ - Q_- \right) + \frac{1}{2} V(a) q_{\text{net}} + \frac{1}{2} E_0 \int z dq_{\text{net}} + \frac{1}{2} E_0 \int z dq_0 \quad (10)$$

where, it must be remembered, both the accumulated charges on either plate ( $Q_+$  and  $Q_-$ ) and the charge distributions ( $q_{\text{net}}$  and  $q_0$ ) depend on the position  $a$  of the conductor. In evaluating the integrals of Equation (10), it is convenient to use the line charge density  $\lambda(z)$ ; thus

$$\int z dq = \int z \lambda(z, a) dz \quad (11)$$

since  $dq = \sigma(z, a) \underline{k} \cdot \underline{dA} = \lambda(z, a) dz$ .

The energy-balance equations are obtained from considering the energy flow to and from the system as shown in Figure 2.

$$Q_{\text{ext.}} V_o = \xi_{\text{electrostatic}} + W \quad (12)$$

The energy flow into the system is simply the energy supplied by the battery: the charge transferred through the external circuit times the potential difference,  $Q_{\text{ext.}} V_o$ . The output energy, labeled here simply as  $W$ , may, in general, be of several forms; for example, kinetic energy of the conductor(s), work done by the system, or dissipative as might result from motion through a viscous medium or sparking between elements in close proximity.

The relationship between  $Q_{\text{ext.}}$  and the charges accumulated on the plates,  $Q_-$  and  $Q_+$ , is established in the following way. By Gauss's law,

$$\phi_- = Q_- \quad \text{and} \quad \phi_+ = Q_+, \quad (13)$$

where  $\phi$  is the inducing flux at either plate as caused by the presence of the conductor(s). It can be shown (see appendix) that, for a point charge  $q$  at position  $Z$ , the fluxes are

$$\phi_- = \left(\frac{1}{2} - Z\right)q \quad \text{and} \quad \phi_+ = \left(\frac{1}{2} + Z\right)q. \quad (14)$$

For a conductor, which can here be described by a line charge  $\lambda(Z)$ , superposition yields

$$\phi_- = \frac{1}{d} \int \left(\frac{1}{2} - Z\right) \lambda(Z) dZ \quad \text{and} \quad (15)$$

$$\phi_+ = \frac{1}{d} \int \left(\frac{1}{2} + Z\right) \lambda(Z) dZ.$$

Since the net flux transfer between the plates (i.e., the displacement flux) is  $\frac{1}{2} (\phi_- + \phi_+)$  and as this displaced flux is, by continuity, equal to the charge transferred through the external current, Equations (13) and (15) lead to

$$Q_{\text{ext.}} = \frac{1}{2} (\phi_- - \phi_+) = \frac{1}{2} (Q_- - Q_+) \quad (16a)$$

$$= - \frac{1}{d} \int_c Z \lambda(Z) dZ \quad (16b)$$

and

$$\xi_{\text{batt.}} = Q_{\text{ext.}} V_o = \frac{1}{2} V_o (Q_- - Q_+). \quad (16c)$$

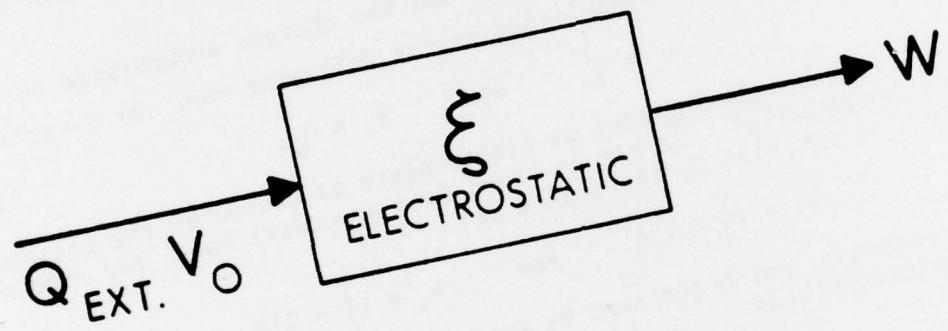


Figure 2. Energy Flow

The energy-flow equation Equation (12) is now separable into two parts by noting that the expression on the right-hand side of Equation (16c) is equal to the first term in Equation (10). Substituting Equations (16) and (10) into (12) then yields the energy-balance equations:

$$\xi_{\text{batt.}} = \xi_{Q_{\pm}} = E_0 \int_C Z \lambda(Z) dZ \quad (17a)$$

$$0 = W + \frac{1}{2} V(a) Q_{\text{net}} + \frac{1}{2} E_0 \int_C Z \lambda_{\text{net}}(Z) dZ + \frac{1}{2} E_0 \int_C Z \lambda_0(Z) dZ \quad (17b)$$

where  $\lambda = \lambda_{\text{net}} + \lambda_0$ .

The interpretation of Equation (17a) is that all the energy supplied by the battery is stored in the form of electrostatic potential energy by charges ( $Q_+$  and  $Q_-$ ) accumulated on the plates and that the magnitude of this energy transfer is determined from  $\lambda(Z)$  by the above integral. The second Equation (17b) shows that the sum of the electrostatic potential energy of the conductor and the external work,  $W$ , is constant; or equivalently, that work is generated by a lowering of the electrostatic energy of the conductor. (NOTE: Any interpretation regarding the partition of electrostatic energy within such a system is not unique. The present interpretation corresponds in a natural way to the development presented here; several other approaches are also possible, and each suggests a different but equivalent way of subpartitioning the electrostatic energy. However, in all other aspects, all approaches are necessarily identical in that Equations (17) result and that no ambiguity exists regarding external quantities.<sup>3</sup> It should be clear from the foregoing that Equations (17) apply to conductors with arbitrary shapes and, by a summation over  $N$  can represent  $N$ -body systems, provided only that the line charges  $\lambda_N$  and the floating potentials  $V_N(a)$  include the effects of all conductor-conductor interactions.

### III. THEOREM FOR NEUTRAL CONDUCTORS

From inspection of the energy-balance equations a very useful theorem is now evident for problems involving neutral conductors:

The change in internal energy of a neutral conducting body is equal to the change in the work  $W$ , and the sum is equal to the loss of potential energy, by the conductor with respect to the uniform field  $E_0$ .

Further, this sum is also equal to the increase in electrical energy supplied externally (by the battery).

<sup>3</sup> W. Panofsky and M. Phillips, "Classical Electricity and Magnetism," Addison-Wiley, 1955, pp. 88-89.

This follows immediately from Equations (17) by setting  $Q_{\text{net}}$  and  $\lambda_{\text{net}}$  to zero,

$$W = \xi_{\text{int.}} = -\frac{1}{2} E_0 \int_C Z \lambda(Z) dZ \quad (18)$$

$$= \frac{1}{2} \xi_{\text{batt.}},$$

where the internal energy  $\xi_{\text{int.}}$  is identical to the assembly energy of the neutral charge distribution on the conductor. In the absence of dissipative forces (e.g., viscous damping, corona, sparking, etc.) the gain in kinetic energy is equal to  $\Delta W$ .

#### IV. ILLUSTRATIVE EXAMPLES

The following problems, involving both neutral and charged bodies, are now presented as examples to illustrate the usefulness of energy-balance in obtaining directly selected information on a variety of otherwise unrelated problems, some of which are not amenable to the usual analytical techniques.

##### A. Problems Involving Neutral Conductors

1. Motion of a Thin Ellipsoidal Conductor on Being Swept into the Uniform Field of a Finite Parallel Plate Capacitor by the Associated Fringing Field. Figure 3 indicates by the superscript (i) the initial conditions and by (f) the parameters at a later time.

$$W^i = KE^i = \xi_{\text{int.}}^i = 0 \quad (19)$$

Equations (17a) and (17b) yield

$$\Delta(KE) = (KE)^f = \frac{1}{2} E_0 \int_C Z \lambda(Z) dZ. \quad (20)$$

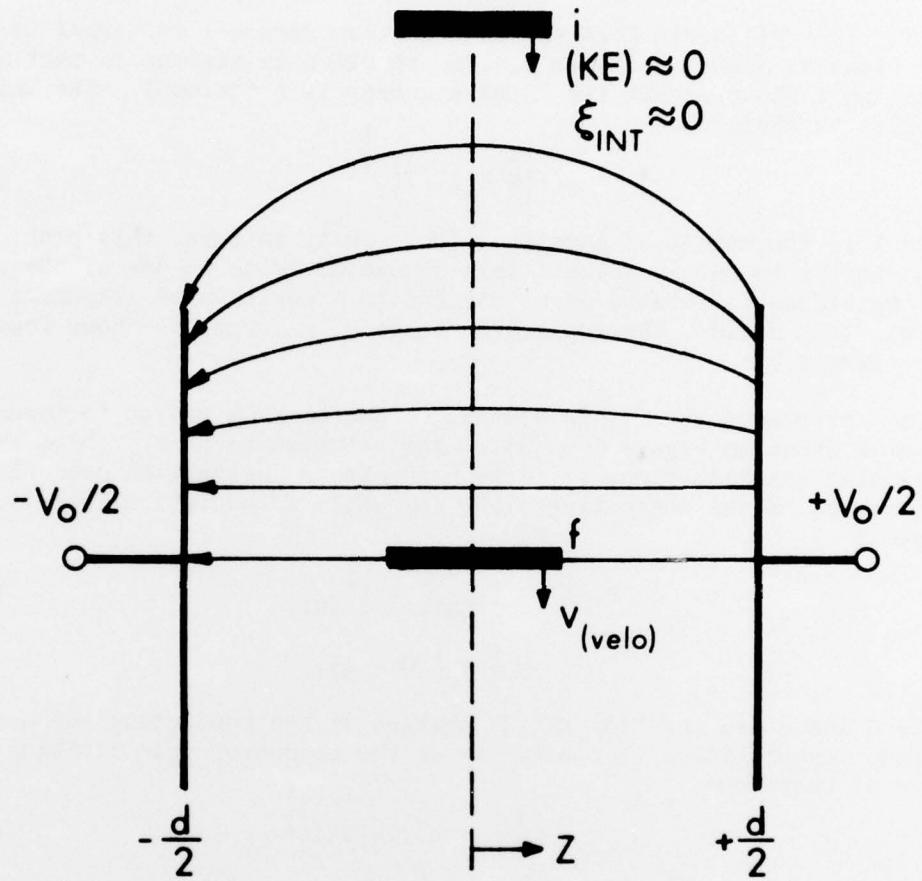
The line charge density is given by

$$\lambda = 8qZ/L^2, \quad (21)$$

where  $q$ , the "separated" charge with corrections for the presence of conducting planes, is determined elsewhere.<sup>4</sup> Integration of Equation (20) leads to the result,

$$\Delta\xi_{\text{batt.}} = 2(\Delta\xi_{\text{int.}}) = 2(\Delta KE) = \frac{2}{3} E_0 qL, \quad (22)$$

<sup>4</sup>W. Bucher, "Electrostatic Forces on Conductors Moving between Conducting Plates," BRL Report in print.



$$\Delta \xi_{\text{BATT}} = 2(\Delta \xi_{\text{INT}}) = 2(\Delta KE) = \frac{2}{3} E_o q L$$

$$v = \left( \frac{2}{3} E_o q L / m \right)^{\frac{1}{2}}$$

Figure 3. Neutral Conductor in Fringing Field

where the kinetic energy is due solely to translation with velocity  $v$  along the median plane,

$$v = \left[ \frac{2}{3} E_0 q L / m \right]^{\frac{1}{2}} \quad (23)$$

2. Thin Ellipsoid Rotated from Position Parallel to Normal to Median Plane as Shown in Figure 4. The solution is similar to that in paragraph 1 above except the kinetic energy is rotational. The angular velocity is therefore

$$\omega = \left[ \frac{2}{3} E_0 q L / I \right]^{\frac{1}{2}} \quad (24)$$

where  $I$  is the moment of inertia. (Note that, in turn, this problem is also similar to one in which a thin conductor lying on one of the conducting planes is rotated about one end to a position of alignment with the electric field. The separated charge  $q$ , however, is about four times larger.)

3. Displacement of Thin Neutral Conductor from Median to Conducting Plane as Shown in Figure 5. (Note, the solution to this problem is of particular interest since it corresponds to an integration over the near-region of the conducting plane for which no analytical solution exists.)

$$\begin{aligned} \Delta(KE) &= \xi_{\text{int.}}^f - \xi_{\text{int.}}^i \\ &= \frac{1}{3} E_0 L (Q - q), \end{aligned} \quad (25)$$

where  $Q$  and  $q$  are the "separated" charges at the conducting and median planes, respectively. The velocity of the conductor upon striking the plate is therefore

$$v = \left[ 2E_0 L (Q - q) / 3m \right]^{\frac{1}{2}} \quad (26)$$

and the charge transferred in the external circuit is

$$Q_{\text{ext.}} = 2L(Q - q) / 3d \quad (27)$$

While the charges  $Q$  and  $q$  can be calculated as previously outlined, the approximation  $Q \approx 4q$  is of sufficient accuracy for most cases of interest.

#### B. Problems Involving Net Charges

1. The determination of the kinetic energy acquired by a thin conductor which carries a charge  $Q_{\text{net}}$ , acquired by contacting one plate, and then moves to the opposite plate is easily carried out by substitution into Equations (17a) and (17b), and will not therefore be presented here. However, one important point concerning the change in the charge

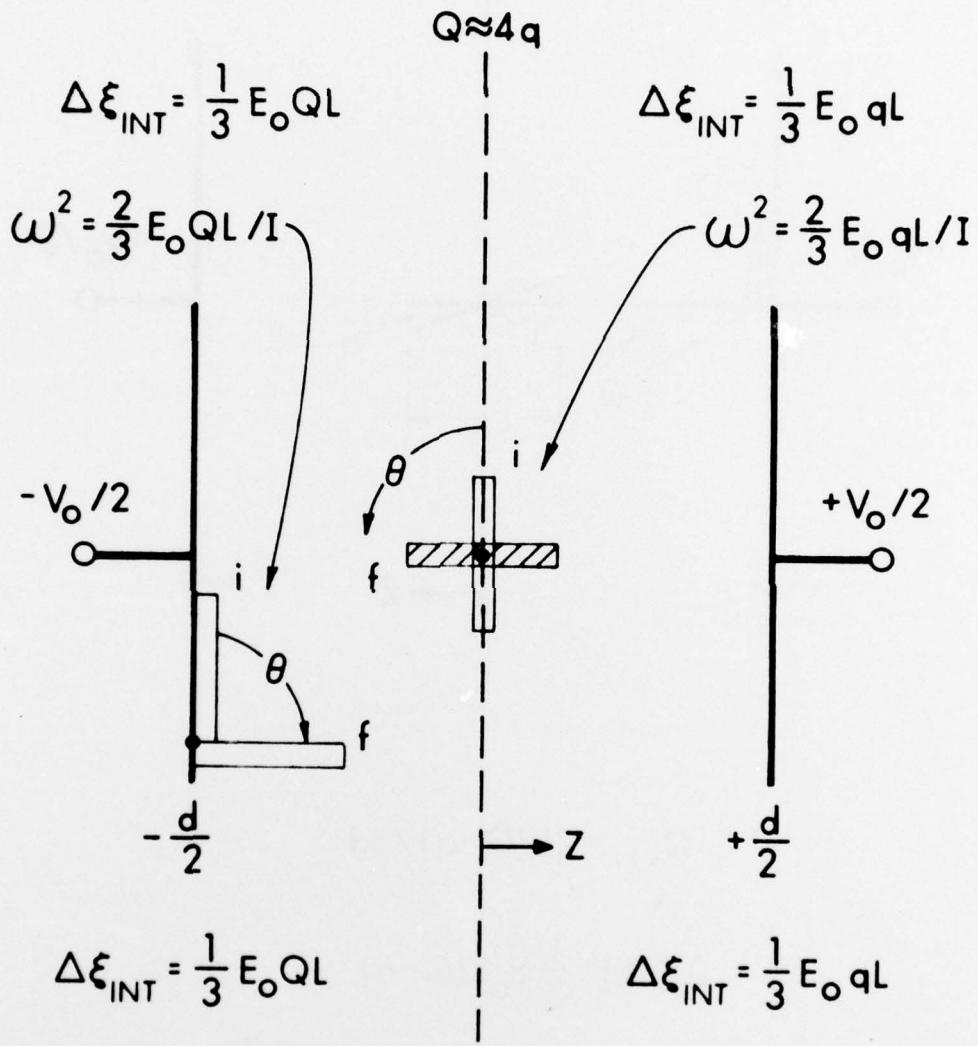
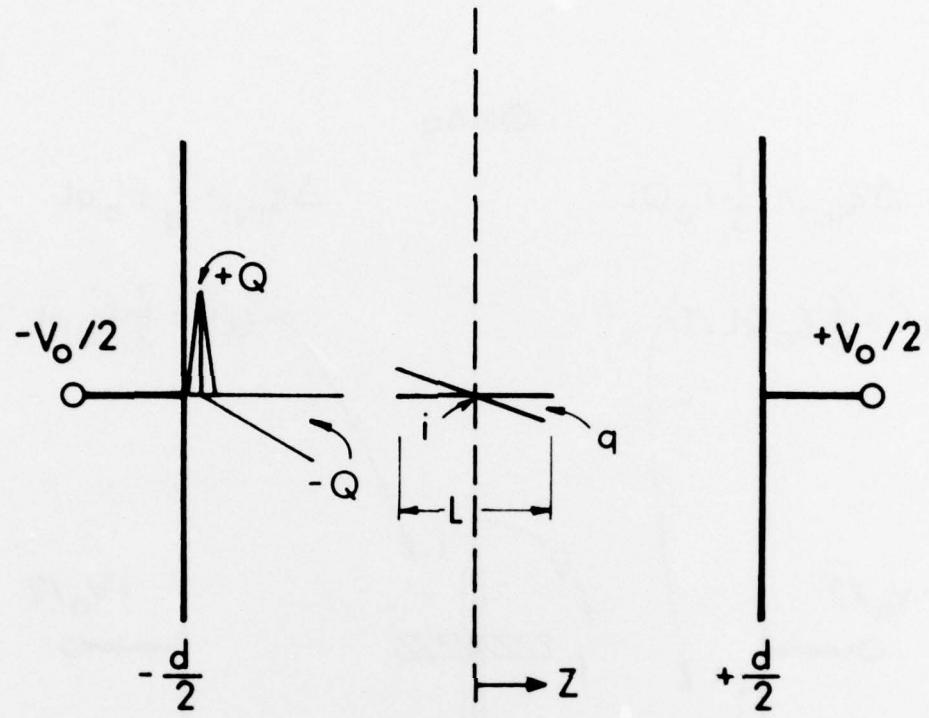


Figure 4. Rotational Motion



$$Q_{EXT} = 2L(Q-q)/3d$$

$$\Delta (KE) = \frac{1}{3} E_o L (Q-q)$$

$$v = \left[ 2E_o L (Q-q)/3m \right]^{\frac{1}{2}}$$

Figure 5. Horizontal Translational Motion

distribution (and, hence, internal energy) during this traversal is noted: The charge distribution undergoes only very small readjustment between the time of leaving the one plate which it has contacted and approaching to within very small distances from the other plate (as indicated in Figure 6). The floating potential, of course, varies more nearly linearly. These variations in the charge distribution are in contrast to those of a neutral body, which undergoes rapid change as either plate is approached. The magnitude of such nonsymmetrical effects as energy transfer (e.g., external circuit current and kinetic energy) can be determined by comparing the results from the energy equations when applied to a charged conductor located at the median plane and at contact on each plane. The required charge distributions at each location were determined previously<sup>4</sup>; only the floating potential at the median plane is required for input into Equations (17). This is found by using the classical solution for an isolated, charged ellipsoid and then applying the method of images to determine the correction due to the conductive plane.

2. While Equations (17) apply only to conductors, they can be modified to include the presence of point charges. The necessary changes will be apparent from the following example from which the kinetic energy of a point charge is determined.

Firstly, the self-energy of a point charge is constant and, as mentioned earlier, was therefore omitted. Secondly, the absence of self-energy causes the last two terms of Equation (17b) to be increased by a factor of two. However, the second integral is necessarily zero for a point charge. Thirdly, the "floating" potential,  $V(a)$  for a point charge must be interpreted as the field due solely to the infinite set of images. Therefore, Equations (17) become

$$\xi_{\text{batt.}} = - E_0 Z q \quad (28)$$

$$0 = W + \frac{1}{2} q \phi(X) + E_0 Z q$$

where  $X = Z + \frac{1}{2} d$  and  $\phi$  is the image potential:

$$\phi(X) = \left( -q/16\pi\epsilon_0 d \right) \left( \frac{1}{X} + 2 \sum_{k=1}^{\infty} \frac{1}{k} \frac{\lambda_k^2}{1-\lambda_k^2} \right),$$

where  $\lambda_k = X/k$ . The kinetic energy, etc., follow simply and directly from Equation (25) differences. Figure 7 shows the potential  $\phi$  for a point charge.

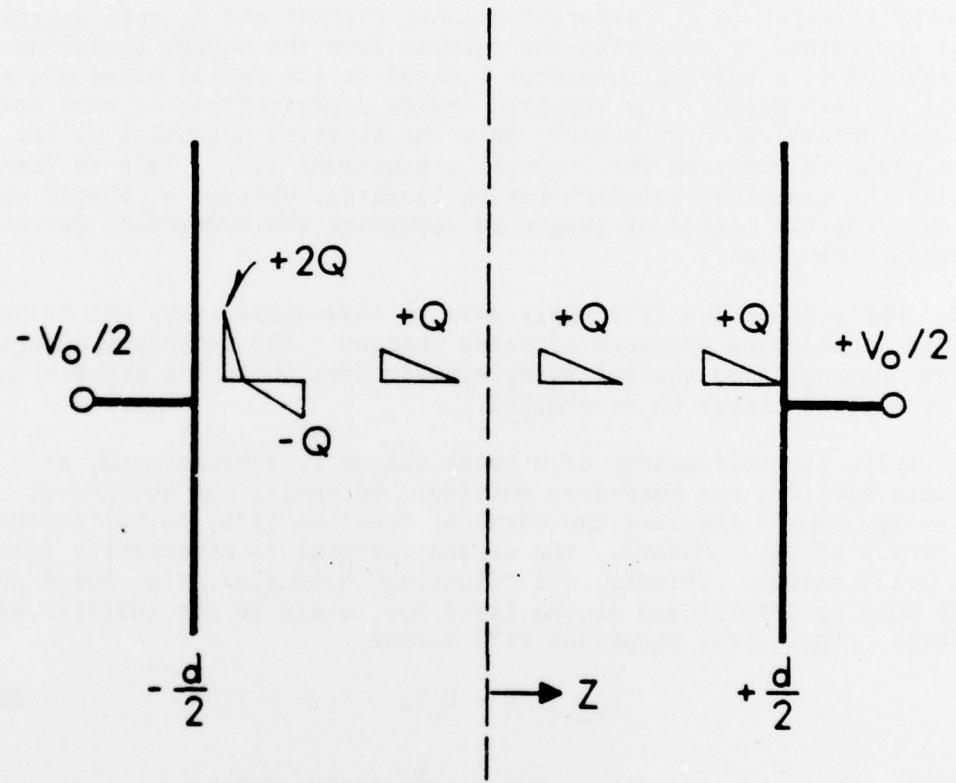
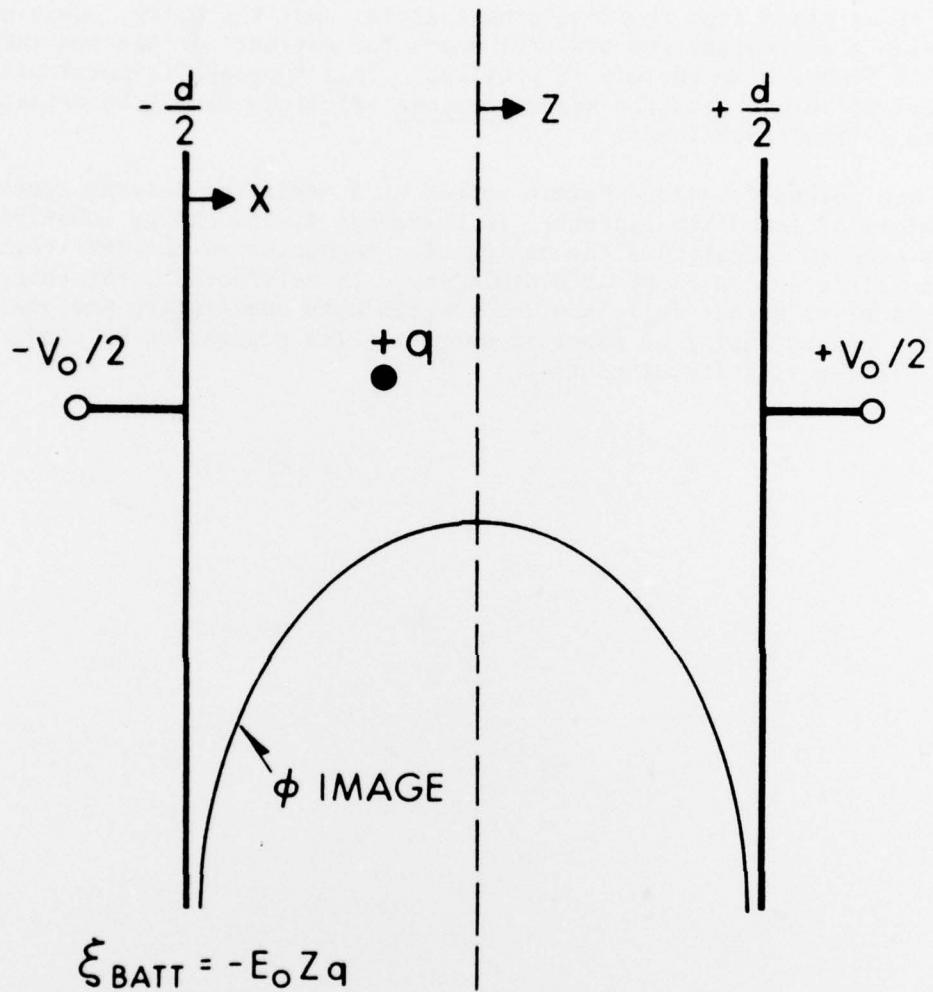


Figure 6. Charged Conductor



$$\xi_{\text{BATT}} = -E_0 Z q$$

$$0 = W + \frac{1}{2} q \phi(x) + E_0 Z q$$

$$\phi(x) = \left( -q / 16\pi\epsilon_0 d \right) \left( \frac{1}{x} + 2 \sum_1^{\infty} \frac{1}{k} \frac{\lambda_k^2}{1 - \lambda_k^2} \right)$$

Figure 7. Point Charge

## V. SUMMARY

It is clear from the foregoing examples that the energy equations provide a convenient and powerful means for extracting selected information from a wide variety of problems. This approach is particularly important in problems for which complete solutions cannot be obtained by analytical techniques.

The method described herein served as a basis for solving several problems of immediate concern. In Reference 4, the energy equations were used to investigate the motion of a conductor in the near-region of an electrode, a point of singularity. In Reference 2, the energy approach was successfully applied to gain both qualitative and quantitative understanding of electric current pulse generation by conductors in the vicinity of contact.

APPENDIX

It is necessary to determine the induced charge distribution, or equivalently the incident flux distribution, on either of two parallel conducting planes due to a unit point charge located between the planes. This problem is most easily solved by the method of images; an infinite set of image charges is generated by multiple reflections at each plane. It is interesting to consider first the solution resulting directly from the infinite summation of images by ordered pairs. The result of this approach is the non-convergent sequence

$$\phi_N = \frac{1}{2} + \frac{1}{2} (-1)^N, \quad (1)$$

where  $\phi_N$  is the flux incident on one plane due to all image pairs up to and including order  $N$ . The explanation of this paradox involves the proper ordering of two limiting processes: the infinite extent of the conducting planes and the infinite set of image charges. The boundary conditions are met only if the later limit is taken first.

The above paradox is thus easily resolved by considering first only that flux incident on a circular area of radius  $Z$ . Then, after summing over the infinite set of images, the limit with respect to  $Z$  can be taken. The above procedure for the first step yields for the flux within the circular area

$$F(X, Z) = 1 - \frac{X}{X^2 + Z^2} + \sum_{N=1}^{\infty} \frac{2N-X}{[Z^2 + (2N-X)^2]^{\frac{1}{2}}},$$

$$- \sum_{N=1}^{\infty} \frac{2N+X}{[Z^2 + (2N+X)^2]^{\frac{1}{2}}} \quad (2)$$

where  $X$  (the distance of the unit charge from the plane) and  $Z$  are in units of  $d$ , the plate separation. A plot of Equation (2) as shown in Figure 8. The second limiting procedure yields

$$\phi(X) = \lim_{Z \rightarrow \infty} F(X, Z) \quad (3)$$

$$= 1 - X \quad \text{in dimensionless units}$$

or

$$\phi(X) = (1 - X/d)q.$$

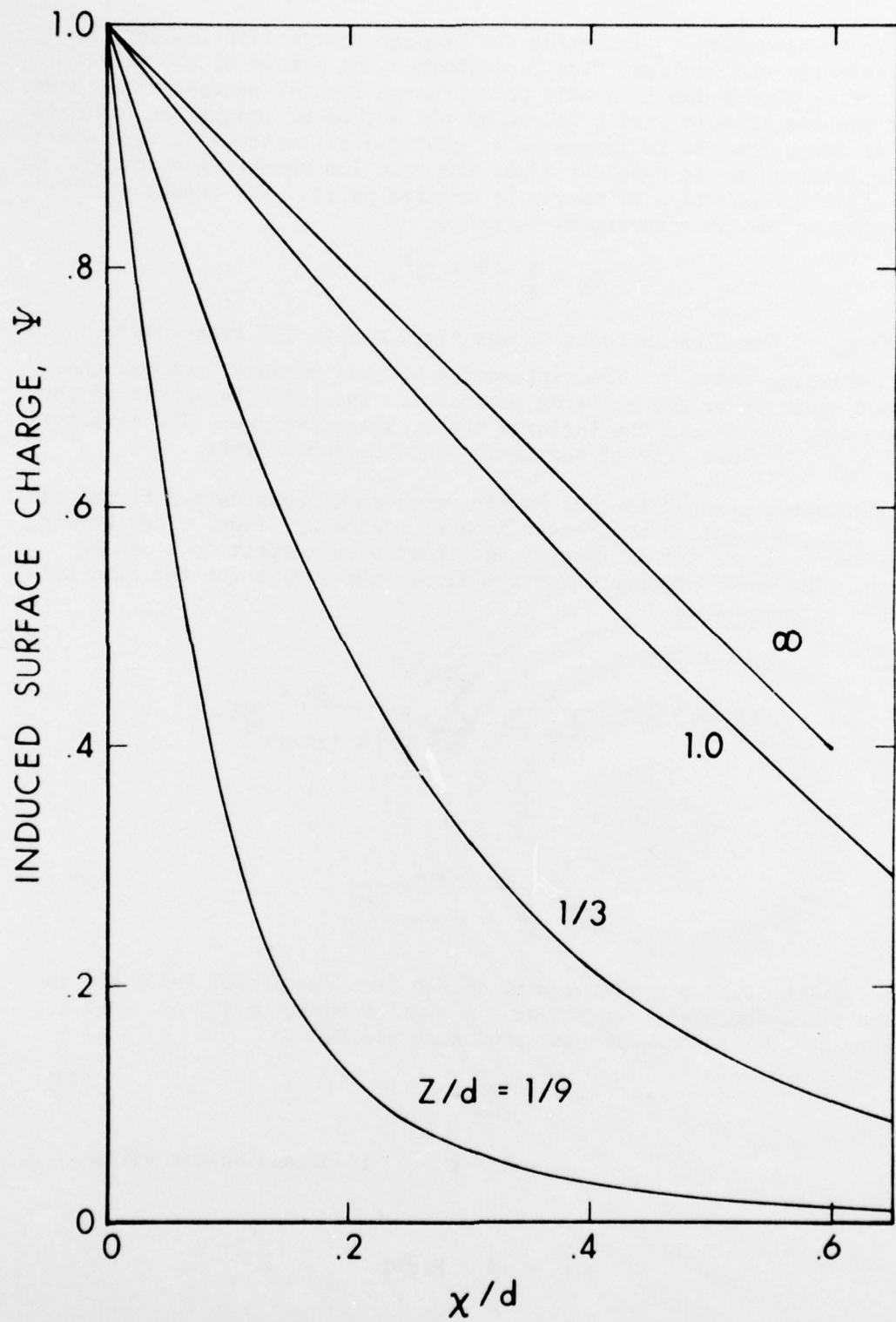


Figure 8. Induced Surface Charge

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